1 POINT FOR EACH MARKED ITEM
UNLESS OTHERWISE NOTED

[2][a] $r = -3 + \cos\theta$ $\frac{1}{2}$ $\left|\frac{-3}{1}\right| = 3 \geq 2 \rightarrow$ CONVEX LIMAÇON $\frac{1}{2}$

NO POINTS WITHOUT $\frac{1}{2}$ NO POINTS WITHOUT

NO PARTIAL CREDIT FOR "LIMAÇON" ONLY

[b] $-3+\cos\theta=0 \longrightarrow \cos\theta=3>1 \longrightarrow$ NO SUCH θ

$r=-3+\cos\theta$ DOES NOT GO THROUGH POLE

SO, NO INTERSECTION AT POLE $\frac{1}{2}$

$-3+\cos\theta=1-3\cos\theta \longrightarrow \cos\theta=1 \longrightarrow \theta=0, 2\pi \longrightarrow r=-2$

INTERSECTION AT $(-2, 0)$ AND $(-2, 2\pi)$ ON BOTH GRAPHS

$-r=-3+\cos(\pi+\theta) \longrightarrow -r=-3-\cos\theta \longrightarrow r=3+\cos\theta$

$3+\cos\theta=1-3\cos\theta \longrightarrow \cos\theta=-\frac{1}{2} \longrightarrow \theta=\frac{2\pi}{3} \in [0,\pi]$

$r=-3+\cos(\pi+\frac{2\pi}{3})=-3+\cos\frac{5\pi}{3}=-3+\frac{1}{2}=-\frac{5}{2}$

$r=1-3\cos\frac{2\pi}{3}=1-3(-\frac{1}{2})=\frac{5}{2}$

INTERSECTION AT $(-\frac{5}{2}, \frac{5\pi}{3})$ ON $r=-3+\cos\theta$

$\frac{1}{2}$ $(\frac{5}{2}, \frac{2\pi}{3})$ ON $r=1-3\cos\theta$

$-r=1-3\cos(\pi+\theta) \longrightarrow -r=1+3\cos\theta \longrightarrow r=-1-3\cos\theta$

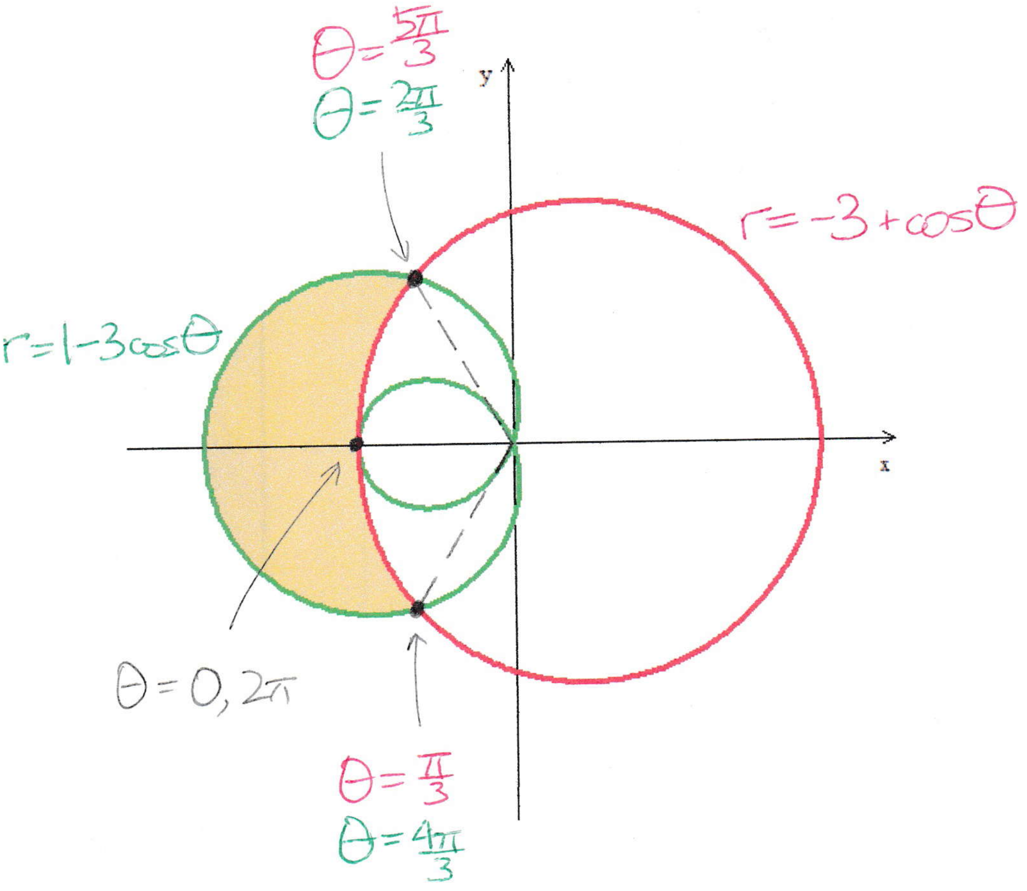
$-3+\cos\theta=-1-3\cos\theta \longrightarrow \cos\theta=\frac{1}{2} \longrightarrow \theta=\frac{\pi}{3} \in [0,\pi]$

$r=-3+\cos\frac{\pi}{3}=-3+\frac{1}{2}=-\frac{5}{2}$

$r=1-3\cos(\pi+\frac{\pi}{3})=1-3\cos\frac{4\pi}{3}=1-3(-\frac{1}{2})=\frac{5}{2}$

INTERSECTION AT $\frac{1}{2}$ $(-\frac{5}{2}, \frac{\pi}{3})$ ON $r=-3+\cos\theta$

$(\frac{5}{2}, \frac{4\pi}{3})$ ON $r=1-3\cos\theta$


θ = 5π/3
θ = 2π/3
y
r = −3 + cosθ
r = 1 − 3cosθ
x
θ = 0, 2π
θ = π/3
θ = 4π/3

[c] $$\frac{dy}{dx} = \frac{[(1-3\cos\theta)\sin\theta]'}{[(1-3\cos\theta)\cos\theta]'} = \frac{\boxed{(3\sin\theta)\sin\theta + (1-3\cos\theta)\cos\theta}\ 1\tfrac{1}{2}}{\boxed{(3\sin\theta)\cos\theta - (1-3\cos\theta)\sin\theta}\ 1\tfrac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{\boxed{\theta=\frac{2\pi}{3}}\ \frac{1}{2}} = \frac{(3\cdot\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2}) + (1-3(-\frac{1}{2}))(-\frac{1}{2})}{(3\cdot\frac{\sqrt{3}}{2})(-\frac{1}{2}) - (1-3(-\frac{1}{2}))(\frac{\sqrt{3}}{2})} = \boxed{\frac{\frac{9}{4}-\frac{5}{4}}{-\frac{3\sqrt{3}}{4}-\frac{5\sqrt{3}}{4}}} = \boxed{\frac{1}{-2\sqrt{3}} \text{ OR } \frac{-\sqrt{3}}{6}}$$

$\frac{1}{2}$

$$(x,y) = \left(\tfrac{5}{2}\cos\tfrac{2\pi}{3}, \tfrac{5}{2}\sin\tfrac{2\pi}{3}\right) = \left(\tfrac{5}{2}(-\tfrac{1}{2}), \tfrac{5}{2}(\tfrac{\sqrt{3}}{2})\right) = \left(-\tfrac{5}{4}, \tfrac{5\sqrt{3}}{4}\right)$$

$$y - \frac{5\sqrt{3}}{4} = -\frac{\sqrt{3}}{6}\left(x+\frac{5}{4}\right)$$

[d] $r\sin\theta - \frac{5\sqrt{3}}{4} = -\frac{\sqrt{3}}{6}\left(r\cos\theta + \frac{5}{4}\right)$

$$r\sin\theta - \frac{5\sqrt{3}}{4} = -\frac{\sqrt{3}}{6}r\cos\theta - \frac{5\sqrt{3}}{24}$$

$$r\sin\theta + \frac{\sqrt{3}}{6}r\cos\theta = \frac{5\sqrt{3}}{4} - \frac{5\sqrt{3}}{24} = \frac{30\sqrt{3} - 5\sqrt{3}}{24} = \frac{25\sqrt{3}}{24}$$

$$r\left(\sin\theta + \frac{\sqrt{3}}{6}\cos\theta\right) = \frac{25\sqrt{3}}{24}$$

MUST HAVE "r ="

$$r = \frac{25\sqrt{3}}{24\left(\sin\theta + \frac{\sqrt{3}}{6}\cos\theta\right)} = \frac{25\sqrt{3}}{24\sin\theta + 4\sqrt{3}\cos\theta}$$

[e] $\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}\underline{(1-3\cos\theta)^2\,d\theta} - \frac{1}{2}\int_{0}^{\frac{\pi}{3}}\underline{(-3+\cos\theta)^2\,d\theta} - \frac{1}{2}\int_{\frac{5\pi}{3}}^{2\pi}\underline{(-3+\cos\theta)^2\,d\theta}$

$1\frac{1}{2}$ $1\frac{1}{2}$

1 POINT FOR "$\frac{1}{2}$" IN ALL INTEGRALS TOGETHER
IE. NO POINTS IF ANY "$\frac{1}{2}$" MISSING

* NO POINTS FOR "$(1-3\cos\theta)^2\,d\theta$"

OR "$(-3+\cos\theta)^2\,d\theta$"

IF LIMITS OF INTEGRATION <u>DON'T</u> MATCH INTEGRAND

OR IF INTEGRAL IS <u>NOT</u> SUBTRACTED FOR "$(-3+\cos\theta)^2\,d\theta$"

ALTERNATE ANSWER:

$$\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(1-3\cos\theta)^2\,d\theta - \left[\frac{1}{2}\int_{0}^{2\pi}(-3+\cos\theta)^2\,d\theta - \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}}(-3+\cos\theta)^2\,d\theta\right]$$